Versor Cheat Sheet

versor.mat.ucsb.edu

February 4, 2013

Graphic Symbol	Geometric State	Grade(s)	Algebraic Form	Abbr.
	Scalar	0	α	`Sca`
	Vector	1	$\boldsymbol{a} = \alpha e_1 + \beta e_2 + \gamma e_3$	`Vec`
	Bivector	2	$\boldsymbol{B} = \boldsymbol{a} \wedge \boldsymbol{b}$	`Biv`
	Trivector	3	$\boldsymbol{I}_3 = \boldsymbol{a} \wedge \boldsymbol{b} \wedge \boldsymbol{c}$	`Tri`
	Point	1	$p = o + \boldsymbol{a} + \frac{1}{2}\boldsymbol{a}^2\infty$	`Pnt`
	Point Pair	2	$\tau = p_a \wedge p_b$	`Par`
	Circle	3	$\kappa = p_a \wedge p_b \wedge p_c$	`Cir`
	Sphere	4	$\Sigma = p_a \wedge p_b \wedge p_c \wedge p_d$	`Sph`
	Flat Point	2	$\Phi = p \wedge \infty$	`Flp`
	Line	3	$\Lambda = p_a \wedge p_b \wedge \infty$	`Lin`
	Dual Line	2	$\lambda = \boldsymbol{B} + \boldsymbol{d}\infty$	`Dll`
	Plane	4	$\Pi = p_a \wedge p_b \wedge p_c \wedge \infty$	`Pln`
	Dual Plane	1	$\pi = \boldsymbol{n} + \delta\infty$	`Dlp`
	Minkowski Plane	2	$E = o \wedge \infty$	`Mnk`
	Direction Vector	2	$\boldsymbol{t}\infty$	`Drv`
	Direction Bivector	3	$\boldsymbol{B}\infty$	`Drb`
	Direction Trivector	4	$\boldsymbol{I}_3\infty$	`Drt`
	Tangent Vector	2	$o\boldsymbol{t}$	`Tnv`
	Tangent Bivector	3	$o\boldsymbol{B}$	`Tnb`
	Tangent Trivector	4	$o\boldsymbol{I}_3$	`Tnt`
	Rotor	0, 2	$\mathscr{R} = e^{-\frac{\theta}{2}\boldsymbol{B}} = cos\frac{\theta}{2} - sin\frac{\theta}{2}\boldsymbol{B}$	`Rot`
	Translator	0, 2	$\mathscr{T} = e^{\frac{\boldsymbol{d}}{2}\infty} = 1 - \frac{\boldsymbol{d}}{2}\infty$	`Trs`
	Motor	0, 2, 4	$\mathscr{M} = e^{\boldsymbol{B}+\boldsymbol{d}\infty}$	`Mot`
	Dilator	0, 2	$\mathscr{D} = e^{\frac{\lambda}{2}E} = cosh\frac{\lambda}{2} + sinh\frac{\lambda}{2}E$	`Dil`
	Boost	0, 2	$\mathscr{B} = e^{o\boldsymbol{t}} = 1 + o\boldsymbol{t}$	`Trv`

Table 1: Basic elements of conformal geometric algebra and their algebraic constructions. The graphic symbols on the left are introduced to help reference the appendix of operations. Bold symbols represent Euclidean elements, with lowercase letters representing 1-blade vectors as is the custom in geometric algebra texts.

Operation	Operator or Method	Expression	Notation
Geometric Product	`*`	`A * B`	AB
Inner Product	`<=`	`A <= B`	$A \rfloor B$
Outer Product	`^`	`A ^ B`	$A \wedge B$
Commutator	`%`	`A % B`	$A \times B$
Inverse	`!`	`!A`	A^{-1}
Reverse	`~`	`~A`	$\tilde{A}$
Conjugate	`conjugation()`	`A.conjugation()`	$\bar{A}$
Involute	`involution()`	`A.involution()`	$\hat{A}$

Table 2: How to perform basic operations on multivectors in *Versor*

Function	Output	Notation
`Op::sp(A,B) or A.sp(B)`	Spin A by B	$\mathscr{B}A\mathscr{B}^{-1}$
`Op::re(A,B) or A.sp(B)`	Reflect A by B	$\mathscr{B}\hat{A}\mathscr{B}^{-1}$
`Op::rj(A,B)`	Rejection of A from B	$(A \wedge B)B^{-1}$
`Op::pj(A,B)`	Projection of A onto B	$(A \rfloor B)B^{-1}$
`Op::dl(A) or A.dual()`	The Dual of A	A^*
`Op::ud(A) or A.undual()`	The Undual of A	A^{-*}
`Op::dle(A) or A.duale()`	The Euclidean Dual of A	$A^\star$
`Op::ude(A) or A.unduale()`	The Euclidean Undual of A	$A^{-\star}$

Table 3: Some useful functions operating on one or two arguments.

Function	Output	Notation
`Gen::log(Rot r)`	The Bivector Generator of input $\mathscr{R}$	$log(\mathscr{R})$
`Gen::log(Mot m)`	The Dual Line Generator of input $\mathscr{M}$	$log(\mathscr{M})$
`Gen::mot(Dll d)`	The exponential $\mathscr{M}$ of input Dual Line $\boldsymbol{B}+\boldsymbol{d}\infty$	$e^{\boldsymbol{B}+\boldsymbol{d}\infty}$
`Gen::rot(Biv b)`	The exponential $\mathscr{R}$ of input Bivector $\theta \boldsymbol{I}$	$e^{-\frac{\theta}{2}\boldsymbol{I}}$
`Gen:ratio(Vec a, Vec b)`	The Rotor $\mathscr{R}$ that takes input Vec $\boldsymbol{a}$ to input Vec $\boldsymbol{b}$	$\frac{(1+\boldsymbol{ba})}{\sqrt{2(1+a\cdot b)}}$
`Gen::ratio(Dll da, Dll db)`	The Motor $\mathscr{M}$ that takes input Dll A to input Dll B	
`Gen::trs`	The exponential $\mathscr{T}$ of input Direction Vector $\boldsymbol{d}\infty$	$e^{-\frac{\boldsymbol{d}}{2}\infty}$
`Gen::dil`	The exponential $\mathscr{D}$ of input E plane λE	$e^{\frac{\lambda}{2}E}$
`Gen::trv`	The exponential $\mathscr{B}$ of input tangent vector	$e^{\frac{\boldsymbol{ot}}{2}}$
`Gen::mat`	The 4x4 matrix of input Rotor $\mathscr{R}$	$\begin{bmatrix} x_0 & y_0 & z_0 & 0 \\ x_1 & y_1 & z_1 & 0 \\ x_2 & y_2 & z_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
`Gen::aa`	The axis angle rep of input Rotor $\mathscr{R}$	$\begin{bmatrix} \theta & x & y & z \end{bmatrix}$

Table 4: Common functions for dealing with versors and their generators.

Function	Output
`Ro::null`	Point p map of input Vector $\boldsymbol{v}$
`Ro::dls`	Dual Sphere from input Vector $\boldsymbol{v}$ and radius α
`Ro::split1`	Point p of input Point Pair $p \wedge q$
`Ro::split2`	Point q of input Point Pair $p \wedge q$
`Ro::sur`	Dual Sphere Surrounding input
`Ro::cen`	Center point p of input Round
`Ro::car`	Carrier Plane or Line of Input Circle or Point Pair
`Ro::size`	Squared Radius (+ or -) of input Round
`Ro::wt`	Weight α of input Round

Table 5: Common functions for creating and querying round elements

Function	Output
`Fl::car`	Carrier Plane or Line of Input Circle or Point Pair
`Fl::loc`	Point p on input line or plane closest to input point
`Fl::wt`	Weight α of input Flat
`Fl::dir`	Direction of input Flat

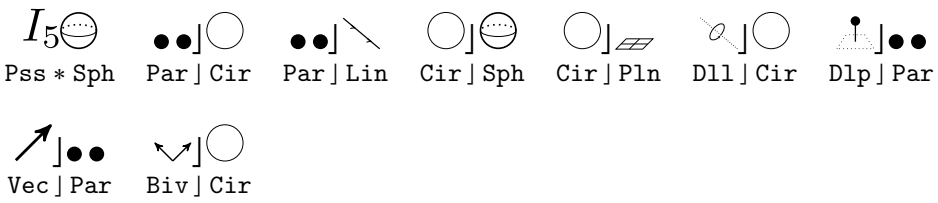
Table 6: Common functions for creating and querying flat elements

Function	Output
`Ta::at`	Tangent to input State at input point p
`Ta::wt`	Weight α of input Tangent

Table 7: Common functions for creating and querying tangent elements

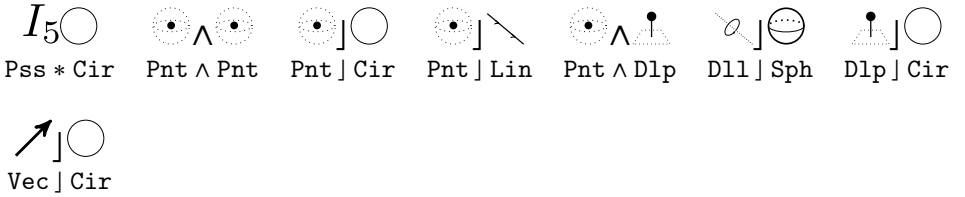
Appendix A: List of Operators By Return Type

Operations that Construct a Point

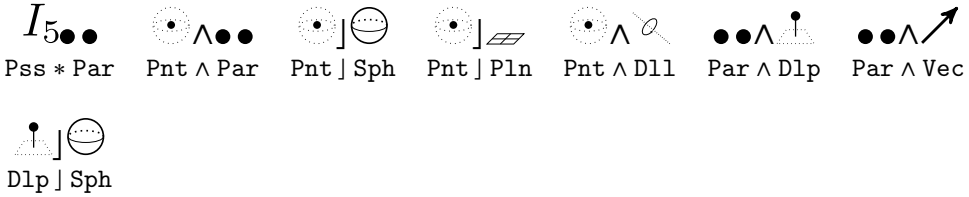


Operations that Construct a Sphere

Operations that Construct a Point Pair



Operations that Construct a Circle



Operations that Construct a Line


Inf ∧ Par


Pss ∗ Dll


Pnt ∧ Drv


Pnt ∧ Flp


Flp ∧ Dlp


Flp ∧ Vec

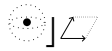

Dlp ⌋ Pln


Vec ⌋ Pln

Operations that Construct a Dual Line


Inf ⌋ Cir


Pss ∗ Lin


Pnt ⌋ Drb


Par ⌋ Drt


Drv ⌋ Sph


Flp ⌋ Sph


Flp ⌋ Pln


Dlp ∧ Dlp


Dlp ∧ Vec

Operations that Construct a Plane


Inf ∧ Cir


Pss ∗ Dlp


Pnt ∧ Drb


Pnt ∧ Lin


Par ∧ Drv


Par ∧ Flp

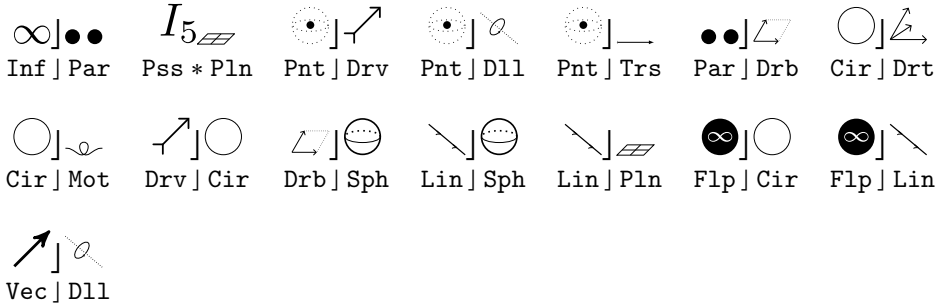

Lin ∧ Dlp


Lin ∧ Vec

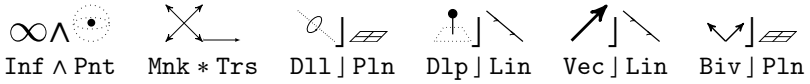

Flp ∧ Dll


Flp ∧ Biv

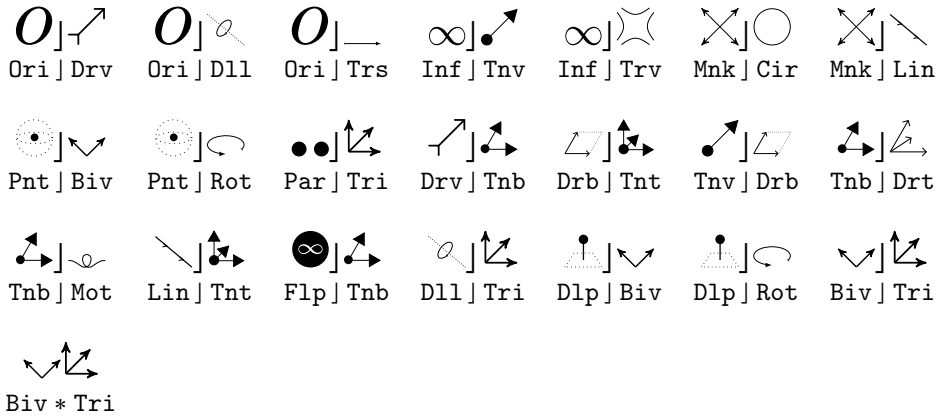
Operations that Construct a Dual Plane



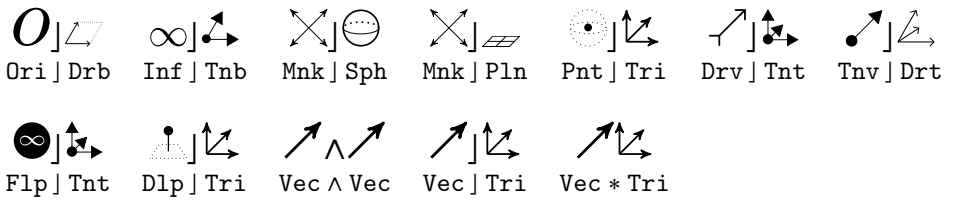
Operations that Construct a Flat Point



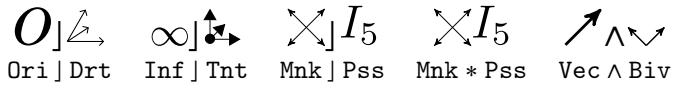
Operations that Construct a Vector



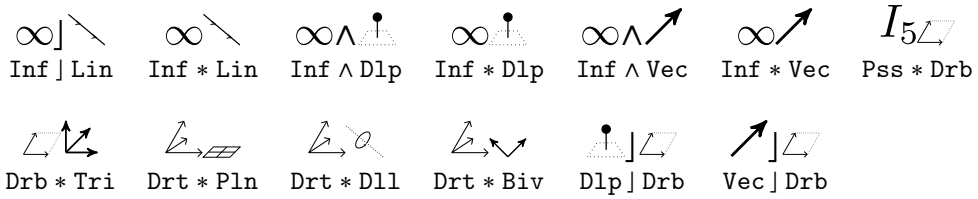
Operations that Construct a Bivector



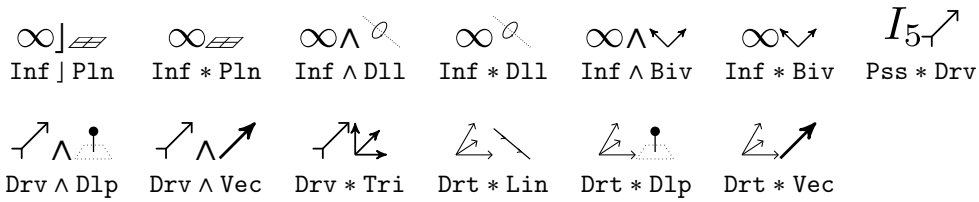
Operations that Construct a Trivector



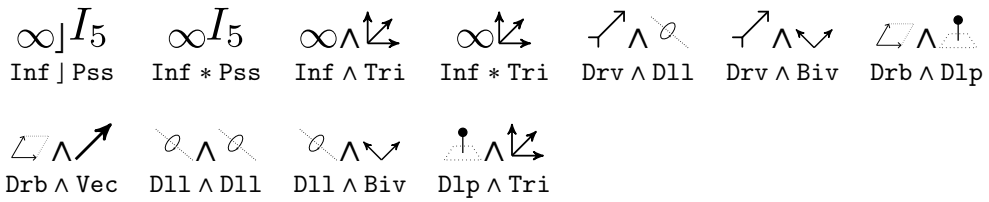
Operations that Construct a Direction Vector



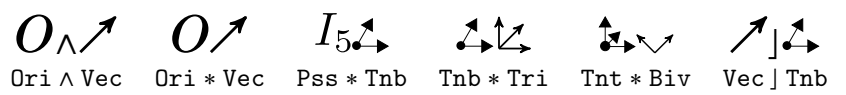
Operations that Construct a Direction Bivector



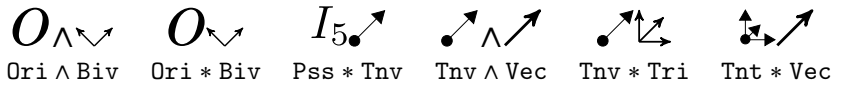
Operations that Construct a Direction Trivector



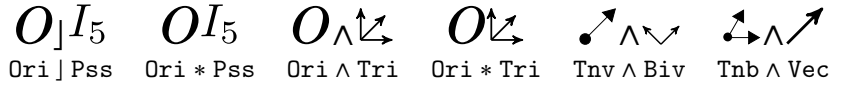
Operations that Construct a Tangent Vector at Origin



Operations that Construct a Tangent Bivector at Origin

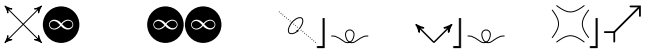


Operations that Construct a Tangent Trivector at Origin



Operations that Construct a Rotor

Operations that Construct a Translator



`Mnk ∗ Flp` `Flp ∗ Flp` `Dll ⌋ Mot` `Biv ⌋ Mot` `Trv ⌋ Drv`

Operations that Construct a Dilator

`Ori ∗ Inf` `Drt ∗ Tnt`

Operations that Construct a Motor

`Lin ∗ Lin` `Dll ∗ Dll` `Dll ∗ Biv` `Dll ∗ Rot` `Rot ∧ Trs` `Rot ∗ Trs`

Operations that Construct a Transversor

`Trs ⌋ Tnv` `Mot ⌋ Tnv`

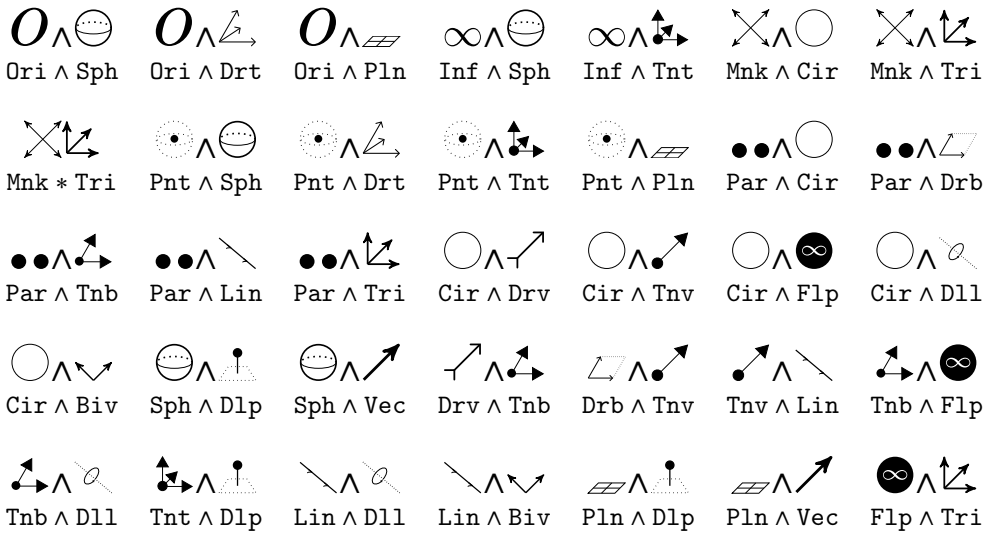
Operations that Construct a Motor Dilator

`Par ∗ Drv` `Cir ∗ Drb` `Dil ∧ Mot` `Dil ∗ Mot`

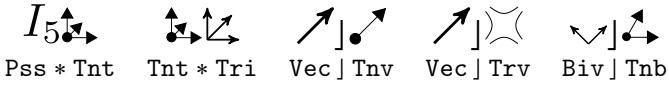
Operations that Construct a Minkowski Plane

`Ori ∧ Inf` `Pss ∗ Tri`

Operations that Construct a Pseudoscalar



Operations that Construct an Origin



Operations that Construct an Infinity